

Chapter 3

Relational Database Languages: Relational Algebra

We first consider only *query* languages.

Relational Algebra: Queries are expressions over operators and relation names.

Relational Calculus: Queries are special formulas of first-order logic with free variables.

SQL: Combination from algebra and calculus and additional constructs. Widely used DML for relational databases.

QBE: Graphical query language.

Deductive Databases: Queries are logical rules.

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RELATIONAL DATABASE LANGUAGES: COMPARISON AND OUTLOOK

Remark:

- Relational Algebra and (safe) Relational Calculus have the same expressive power. For every expression of the algebra there is an equivalent expression in the calculus, and vice versa.
- A query language is called **relationally complete**, if it is (at least) as expressive as the relational algebra.
- These languages are compromises between efficiency and expressive power; they are not computationally complete (i.e., they cannot simulate a Turing Machine).
- They can be embedded into host languages (e.g. C++ or Java) or extended (PL/SQL), resulting in full computational completeness.
- Deductive Databases (Datalog) are more expressive than relational algebra and calculus.

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3.1 Relational Algebra: Computations over Relations

Operations on Tuples – Overview Slide

Let $\mu \in \text{Dup}(\bar{X})$ where $\bar{X} = \{A_1, \dots, A_k\}$.

(Formal definition of μ see Slide 61)

- For $\emptyset \subset \bar{Y} \subseteq \bar{X}$, the expression $\mu[\bar{Y}]$ denotes the **projection** of μ to \bar{Y} .

Result: $\mu[\bar{Y}] \in \text{Dup}(\bar{Y})$ where $\mu[\bar{Y}](A) = \mu(A), A \in \bar{Y}$.

- A **selection condition** α (wrt. \bar{X}) is an expression of the form $A \theta B$ or $A \theta c$, or $c \theta A$ where $A, B \in \bar{X}$, $\text{dom}(A) = \text{dom}(B)$, $c \in \text{dom}(A)$, and θ is a **comparison operator** on that domain like e.g. $\{=, \neq, \leq, <, \geq, >\}$.

A tuple $\mu \in \text{Dup}(\bar{X})$ **satisfies** a selection condition α , if – according to α – $\mu(A) \theta \mu(B)$ or $\mu(A) \theta c$, or $c \theta \mu(A)$ holds.

These (atomic) selection conditions can be combined to formulas by using \wedge, \vee, \neg , and $(,)$.

- For $\bar{Y} = \{B_1, \dots, B_k\}$, the expression $\mu[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k]$ denotes the **renaming** of μ .

Result: $\mu[\dots, A_i \rightarrow B_i, \dots] \in \text{Dup}(\bar{Y})$ where $\mu[\dots, A_i \rightarrow B_i, \dots](B_i) = \mu(A_i)$ for $1 \leq i \leq k$.

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Let $\mu \in \text{Dup}(\bar{X})$ where $\bar{X} = \{A_1, \dots, A_k\}$.

Projection (Reduction to a subset of the attributes)

For $\emptyset \subset \bar{Y} \subseteq \bar{X}$, the expression $\mu[\bar{Y}]$ denotes the **projection** of μ to \bar{Y} .

Result: $\mu[\bar{Y}] \in \text{Dup}(\bar{Y})$ where $\mu[\bar{Y}](A) = \mu(A), A \in \bar{Y}$.

projection to a given set of attributes

Example 3.1

Consider the relation schema $R(\bar{X}) = \text{Continent}(\text{name}, \text{area})$: $\bar{X} = [\text{name}, \text{area}]$

and the tuple $\mu = \boxed{\text{name} \mapsto \text{“Asia”}, \text{area} \mapsto 4.50953e+07}$.

formally: $\mu(\text{name}) = \text{“Asia”}, \mu(\text{area}) = 4.5E7$

projection attributes: Let $\bar{Y} = [\text{name}]$

Result: $\mu[\text{name}] = \boxed{\text{name} \mapsto \text{“Asia”}}$

□

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Again, $\mu \in \text{Tup}(\bar{X})$ where $\bar{X} = \{A_1, \dots, A_k\}$.

Selection (only those tuples that satisfy some condition)

A **selection condition** α (wrt. \bar{X}) is an expression of the form $A \theta B$ or $A \theta c$, or $c \theta A$ where $A, B \in \bar{X}$, $\text{dom}(A) = \text{dom}(B)$, $c \in \text{dom}(A)$, and θ is a **comparison operator** on that domain like e.g. $\{=, \neq, \leq, <, \geq, >\}$.

A tuple $\mu \in \text{Tup}(\bar{X})$ **satisfies** a selection condition α , if – according to α – $\mu(A) \theta \mu(B)$ or $\mu(A) \theta c$, or $c \theta \mu(A)$ holds.

yes/no-selection of tuples (without changing the tuple)

Example 3.2

Consider again the relation schema $R(\bar{X}) = \text{continent}(\text{name}, \text{area})$: $\bar{X} = [\text{name}, \text{area}]$.

Selection condition: $\text{area} > 20000000$.

Consider again the tuple $\mu = \boxed{\text{name} \mapsto \text{“Asia”}, \text{area} \mapsto 4.50953e+07}$.

formally: $\mu(\text{name}) = \text{“Asia”}$, $\mu(\text{area}) = 4.5E7$

check: $\mu(\text{area}) > 20000000$

Result: yes. □

These (atomic) selection conditions can be combined to formulas by using \wedge , \vee , \neg , and $(,)$.

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Let $\mu \in \text{Tup}(\bar{X})$ where $\bar{X} = \{A_1, \dots, A_k\}$.

Renaming (of attributes)

For $\bar{Y} = \{B_1, \dots, B_k\}$, the expression $\mu[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k]$ denotes the **renaming** of μ .

Result: $\mu[\dots, A_i \rightarrow B_i, \dots] \in \text{Tup}(\bar{Y})$ where $\mu[\dots, A_i \rightarrow B_i, \dots](B_i) = \mu(A_i)$ for $1 \leq i \leq k$.

renaming of attributes (without changing the tuple)

Example 3.3

Consider (for a tuple of the table $R(\bar{X}) = \text{encompasses}(\text{country}, \text{continent}, \text{percent})$):

$\bar{X} = [\text{country}, \text{continent}, \text{percent}]$.

Consider the tuple $\mu = \boxed{\text{country} \mapsto \text{“R”}, \text{continent} \mapsto \text{“Asia”}, \text{percent} \mapsto 80}$.

formally: $\mu(\text{country}) = \text{“R”}$, $\mu(\text{continent}) = \text{“Asia”}$, $\mu(\text{percent}) = 80$

Renaming: $\bar{Y} = [\text{code}, \text{name}, \text{percent}]$

Result: a new tuple $\mu[\text{country} \rightarrow \text{code}, \text{continent} \rightarrow \text{name}, \text{percent} \rightarrow \text{percent}] =$

$\boxed{\text{code} \mapsto \text{“R”}, \text{name} \mapsto \text{“Asia”}, \text{percent} \mapsto 80}$ that now fits into the schema $\text{new_encompasses}(\text{code}, \text{name}, \text{percent})$. □

The usefulness of renaming will become clear later ...

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EXPRESSIONS IN THE RELATIONAL ALGEBRA

What is an algebra?

- An algebra consists of a "domain" (i.e., a set of "things"), and a set of operators.
- Operators map elements of the domain to other elements of the domain.
- Each of the operators has a "semantics", that is, a definition how the result of applying it to some input should look like.
- **Algebra expressions** are built over basic constants and operators (inductive definition).

Relational Algebra

- The "domain" consists of all relations (over arbitrary sets of attributes).
- The operators are then used for combining relations, and for describing computations - e.g., in SQL.
- **Relational algebra expressions** are defined inductively over relations and operators.
- Relational algebra expressions define queries against a relational database.

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INDUCTIVE DEFINITION OF EXPRESSIONS

Atomic Expressions - Base Cases of the Inductive Definition

- For an arbitrary attribute A and a constant $c \in \text{dom}(A)$, the **constant relation** $A : \{c\}$ is an algebra expression.

Format: $[A]$

Result relation: $\{\mu\}$ with $\mu = (A \mapsto c)$

A:{c}
A
c

- Given a database schema $\mathbf{R} = \{R_1(\bar{X}_1), \dots, R_n(\bar{X}_n)\}$, every relation name R_i is an algebra expression.

Format of R_i : \bar{X}_i

Result relation (wrt. a given database state S): the relation $S(R_i)$ that is currently stored in the database.

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Structural Induction: Applying an Operator

- takes one or more input relations in_1, in_2, \dots
- produces a result relation out :
 - out has a **format**, depends on the formats of the input relations.
 - out is a relation, i.e., it contains some tuples, depends on the content of the input relations.
- Note: the relational algebra is based on mathematical *set theory*
 \Rightarrow sets do not contain duplicates, i.e., whenever duplicates would occur, they are immediately removed.
 (SQL in contrast is based on *multisets* that can contain duplicates)

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BASE OPERATORS

Let \bar{X}, \bar{Y} formats and $r \in \text{Rel}(\bar{X})$ and $s \in \text{Rel}(\bar{Y})$ relations over \bar{X} and \bar{Y} .

Union

Assume $r, s \in \text{Rel}(\bar{X})$.

Result format of $r \cup s$: \bar{X}

Result relation: $r \cup s = \{\mu \in \text{Dup}(\bar{X}) \mid \mu \in r \text{ or } \mu \in s\}$.

	A	B	C
$r =$	a	b	c
	d	a	f
	c	b	d

	A	B	C
$s =$	b	g	a
	d	a	f

	A	B	C
$r \cup s =$	a	b	c
	d	a	f
	c	b	d
	b	g	a

(note: no duplicates in the result - based on set theory)

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Set Difference

Assume $r, s \in \text{Rel}(\bar{X})$.

Result format of $r \setminus s$: \bar{X}

Result relation: $r \setminus s = \{\mu \in r \mid \mu \notin s\}$.

	<u>A</u>	<u>B</u>	<u>C</u>
$r =$	a	b	c
	d	a	f
	c	b	d

	<u>A</u>	<u>B</u>	<u>C</u>
$s =$	b	g	a
	d	a	f

	<u>A</u>	<u>B</u>	<u>C</u>
$r \setminus s =$	a	b	c
	c	b	d

Projection (Reduction to a subset of the attributes)

Assume $r \in \text{Rel}(\bar{X})$ and $\bar{Y} \subseteq \bar{X}$.

Result format of $\pi[\bar{Y}](r)$: \bar{Y}

Result relation: $\pi[\bar{Y}](r) = \{\mu[\bar{Y}] \mid \mu \in r\}$.

Example 3.4

Continent	
name	area
Europe	10523000
Africa	30221500
Asia	44614500
N. America	24709000
S. America	17840000
Australia	9000000

Let $\bar{Y} = [\text{name}]$

$\mu_1[\text{name}] = \text{name} \mapsto \text{"Europe"}$

$\mu_2[\text{name}] = \text{name} \mapsto \text{"Africa"}$

$\mu_3[\text{name}] = \text{name} \mapsto \text{"Asia"}$

$\mu_4[\text{name}] = \text{name} \mapsto \text{"N.America"}$

$\mu_4[\text{name}] = \text{name} \mapsto \text{"S.America"}$

$\mu_5[\text{name}] = \text{name} \mapsto \text{"Australia"}$

$\pi[\text{name}](\text{Continent})$
name
Europe
Africa
Asia
N.America
S.America
Australia

□

Selection (Reduction of number of tuples by a condition)

Assume $r \in \text{Rel}(\bar{X})$ and a selection condition α over \bar{X} .

Result format of $\sigma[\alpha](r)$: \bar{X}

Result relation: $\sigma[\alpha](r) = \{\mu \in r \mid \mu \text{ satisfies } \alpha\}$.

Example 3.5

Continent	
name	area
Europe	10523000
Africa	30221500
Asia	44614500
N. America	24709000
S. America	17840000
Australia	9000000

Let $\alpha = \text{"area} > 20000000\text{"}$

$\mu_1(\text{area}) > 20000000?$ – no

$\mu_2(\text{area}) > 20000000?$ – yes

$\mu_3(\text{area}) > 20000000?$ – yes

$\mu_4(\text{area}) > 20000000?$ – yes

$\mu_4(\text{area}) > 20000000?$ – no

$\mu_5(\text{area}) > 20000000?$ – no

$\sigma[\text{area} > 20E6](\text{Continent})$	
name	area
Africa	30221500
Asia	44614500
N.America	24709000

□

Renaming (of attributes)

Assume $r \in \text{Rel}(\bar{X})$ with $\bar{X} = [A_1, \dots, A_k]$ and a renaming $[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k]$.

Result format of $\rho[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k](r)$: $[B_1, \dots, B_k]$

Result relation: $\rho[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k](r) = \{\mu[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k] \mid \mu \in r\}$.

Example 3.6

Consider the renaming of the table *encompasses*(country, continent, percent):

$\bar{X} = [\text{country}, \text{continent}, \text{percent}]$

Renaming: $\bar{Y} = [\text{code}, \text{name}, \text{percent}]$

$\rho[\text{country} \rightarrow \text{code}, \text{continent} \rightarrow \text{name}, \text{percent} \rightarrow \text{percent}](\text{encompasses})$		
code	name	percent
R	Europe	20
R	Asia	80
D	Europe	100
⋮	⋮	⋮

□

(Natural) Join (Combining two relations via common attributes)

Assume $r \in \text{Rel}(\bar{X})$ and $s \in \text{Rel}(\bar{Y})$ for arbitrary \bar{X}, \bar{Y} .

Convention: For $\bar{X} \cup \bar{Y}$, as a shorthand, write \overline{XY} .

for two tuples $\mu_1 = [v_1, \dots, v_n]$ and $\mu_2 = [w_1, \dots, w_m]$, $\mu_1\mu_2 := [v_1, \dots, v_n, w_1, \dots, w_m]$.

Result format of $r \bowtie s$: \overline{XY} .

Result relation: $r \bowtie s = \{\mu \in \text{Dup}(\overline{XY}) \mid \mu[\bar{X}] \in r \text{ and } \mu[\bar{Y}] \in s\}$.

Motivation

Simplest Case: $\bar{X} \cap \bar{Y} = \emptyset \Rightarrow$ Cartesian Product $r \bowtie s = r \times s$

$r \times s = \{\mu_1\mu_2 \in \text{Dup}(\overline{XY}) \mid \mu_1 \in r \text{ and } \mu_2 \in s\}$.

$r =$	<table border="1" style="border-collapse: collapse;"> <tr><th>A</th><th>B</th></tr> <tr><td>1</td><td>2</td></tr> <tr><td>4</td><td>5</td></tr> </table>	A	B	1	2	4	5	$s =$	<table border="1" style="border-collapse: collapse;"> <tr><th>C</th><th>D</th></tr> <tr><td>a</td><td>b</td></tr> <tr><td>c</td><td>d</td></tr> <tr><td>e</td><td>f</td></tr> </table>	C	D	a	b	c	d	e	f	$r \bowtie s =$	<table border="1" style="border-collapse: collapse;"> <tr><th>A</th><th>B</th><th>C</th><th>D</th></tr> <tr><td>1</td><td>2</td><td>a</td><td>b</td></tr> <tr><td>1</td><td>2</td><td>c</td><td>d</td></tr> <tr><td>1</td><td>2</td><td>e</td><td>f</td></tr> <tr><td>4</td><td>5</td><td>a</td><td>b</td></tr> <tr><td>4</td><td>5</td><td>c</td><td>d</td></tr> <tr><td>4</td><td>5</td><td>e</td><td>f</td></tr> </table>	A	B	C	D	1	2	a	b	1	2	c	d	1	2	e	f	4	5	a	b	4	5	c	d	4	5	e	f
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Example 3.7 (Cartesian Product of Continent and Encompasses)

The cartesian product combines everything with everything, not only “meaningful” combinations:

Continent × encompasses				
name	area	continent	country	percent
Europe	10523000	Europe	D	100
Europe	10523000	Europe	R	20
Europe	10523000	Asia	R	80
Europe	10523000	:	:	:
Africa	30221500	Europe	D	100
Africa	30221500	Europe	R	20
Africa	30221500	Asia	R	80
Africa	30221500	:	:	:
Asia	44614500	Europe	D	100
Asia	44614500	Europe	R	20
Asia	44614500	Asia	R	80
Asia	44614500	:	:	:
:	:	:	:	:

Back to the Natural Join

General case $\bar{X} \cap \bar{Y} \neq \emptyset$: shared attribute names constrain the result relation.

Again the definition: $r \bowtie s = \{\mu \in \text{Dup}(\overline{XY}) \mid \mu[\bar{X}] \in r \text{ and } \mu[\bar{Y}] \in s\}$.

(Note: this implies that the tuples $\mu_1 := \mu[\bar{X}] \in r$ and $\mu_2 := \mu[\bar{Y}] \in s$ coincide in the shared attributes $\bar{X} \cap \bar{Y}$)

Example 3.8

Consider *encompasses*(country,continent,percent) and *isMember*(organization,country,type):

<i>encompasses</i>			<i>isMember</i>		
country	continent	percent	organization	country	type
R	Europe	20	EU	D	member
R	Asia	80	UN	D	member
D	Europe	100	UN	R	member
:	:	:	:	:	:

$$\begin{aligned} \text{encompasses} \bowtie \text{isMember} = \{ \mu \in \text{Dup}(\text{country, cont, perc, org, type}) \mid \\ \mu[\text{country, cont, perc}] \in \text{encompasses} \text{ and } \mu[\text{org, country, type}] \in \text{isMember} \} \end{aligned}$$

□

Example 3.8 (Continued)

$$\begin{aligned} \text{encompasses} \bowtie \text{isMember} = \{ \mu \in \text{Dup}(\text{country, cont, perc, org, type}) \mid \\ \mu[\text{country, cont, perc}] \in \text{encompasses} \text{ and } \mu[\text{org, country, type}] \in \text{isMember} \} \end{aligned}$$

start with $(R, \text{Europe}, 20) \in \text{encompasses}$.

check which tuples in *isMember* match:

$(UN, R, \text{member}) \in \text{isMember}$ matches:

result: $(R, \text{Europe}, 20, UN, \text{member})$ belongs to the result.

(some more matches ...)

continue with $(R, \text{Asia}, 80) \in \text{encompasses}$.

$(UN, R, \text{member}) \in \text{isMember}$ matches:

result: $(R, \text{Asia}, 80, UN, \text{member})$ belongs to the result.

(some more matches ...)

continue with $(D, \text{Europe}, 100) \in \text{encompasses}$.

$(EU, D, \text{member}) \in \text{isMember}$ matches:

result: $(D, \text{Europe}, 100, EU, \text{member})$ belongs to the result.

$(UN, D, \text{member}) \in \text{isMember}$ matches:

result: $(D, \text{Europe}, 100, UN, \text{member})$ belongs to the result.

(some more matches ...)

□

Example 3.8 (Continued)

Result:

<i>encompasses</i> ⋈ <i>isMember</i>				
<i>country</i>	<i>continent</i>	<i>percent</i>	<i>organization</i>	<i>type</i>
<i>R</i>	<i>Europe</i>	<i>20</i>	<i>UN</i>	<i>member</i>
<i>R</i>	<i>Europe</i>	<i>20</i>	:	:
<i>R</i>	<i>Asia</i>	<i>80</i>	<i>UN</i>	<i>member</i>
<i>R</i>	<i>Asia</i>	<i>80</i>	:	:
<i>D</i>	<i>Europe</i>	<i>100</i>	<i>UN</i>	<i>member</i>
<i>D</i>	<i>Europe</i>	<i>100</i>	<i>EU</i>	<i>member</i>
<i>D</i>	<i>Europe</i>	<i>100</i>	:	:
:	:	:	:	:

□

Example 3.9 (and Exercise)

Consider the expression

$Continent \bowtie \rho[country \rightarrow code, continent \rightarrow name, percent \rightarrow percent](encompasses)$

□

Functionalities of the Join

- Combining relations
- Selective functionality: only matching tuples survive
(consider joining cities and organizations on headquarters)

DERIVED OPERATORS

Intersection

Assume $r, s \in \text{Rel}(\bar{X})$.

Then, $r \cap s = \{\mu \in \text{Tup}(\bar{X}) \mid \mu \in r \text{ and } \mu \in s\}$.

Theorem 3.1

Intersection can be expressed by difference: $r \cap s = r \setminus (r \setminus s)$.

□

θ -Join

Combination of **Cartesian Product** and **Selection**:

Assume $r \in \text{Rel}(\bar{X})$, and $s \in \text{Rel}(\bar{Y})$, such that $\bar{X} \cap \bar{Y} = \emptyset$, and $A \theta B$ a selection condition.

$$r \bowtie_{A\theta B} s = \{\mu \in \text{Dup}(\overline{XY}) \mid \mu[\bar{X}] \in r, \mu[\bar{Y}] \in s \text{ and } \mu \text{ satisfies } A\theta B\} = \sigma[A\theta B](r \times s).$$

Equi-Join

θ -join that uses the “=”-predicate.

Example 3.10 (and Exercise)

Consider again Example 3.7:

$\text{Continent} \bowtie \text{encompasses} = \text{Continent} \times \text{encompasses}$ contained tuples that did not really make sense.

$\text{Continent} \bowtie_{\text{continent=name}} \text{encompasses}$ would be more useful.

Furthermore, consider

$\pi[\text{continent}, \text{area}, \text{code}, \text{percent}](\text{Continent} \bowtie_{\text{continent=name}} \text{encompasses})$:

- removes the - now redundant - “name” column,
- is equivalent to the natural join $(\rho[\text{name} \rightarrow \text{continent}](\text{continent})) \bowtie \text{encompasses}$. \square

Semi-Join

- recall: joins combine, but are also selective
- semi-join acts like a selection on a relation r :
selection condition not given as a boolean formula on the attributes of r , but by “looking into” another relation (a subquery)

Assume $r \in \text{Rel}(\bar{X})$ and $s \in \text{Rel}(\bar{Y})$ such that $\bar{X} \cap \bar{Y} \neq \emptyset$.

Result format of $r \ltimes s$: \bar{X}

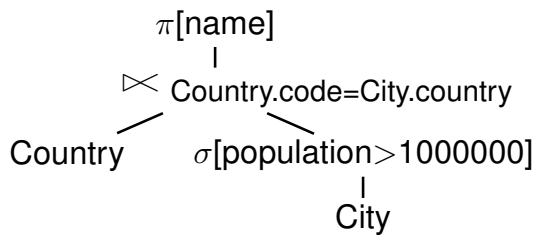
Result relation: $r \ltimes s = \pi[\bar{X}](r \bowtie s)$

The semi-join $r \ltimes s$ does *not* return the join, but checks which tuples of r “survive” the join with s (i.e., “which find a counterpart in s wrt. the shared attributes”):

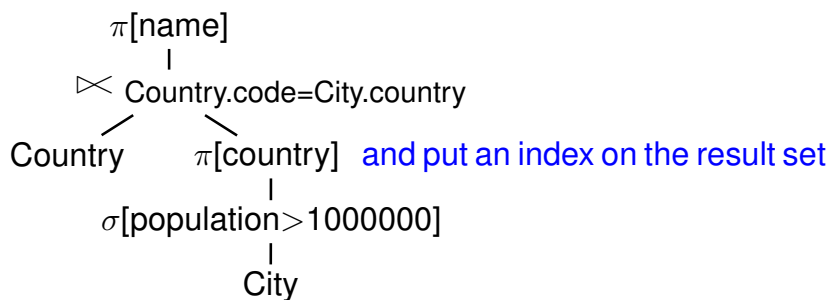
- Used with subqueries: (main query) \ltimes (subquery)
- $r \ltimes s \subseteq r$
- Used for optimizing the evaluation of joins (often in combination with indexes).

Semi-Join: Example

Give the names of all countries where a city with at least 1000000 inhabitants is located:



- Have a short look “inside” the subquery, but don’t actually use it:
- look only if there is a big city in this country.
- “if the country code is in the set of country codes ...”:



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Towards the Outer Join

- The (inner) join is the operator for combining relations

Example 3.11

- *Persons work in divisions of a company, tools are assigned to the divisions:*

Works	
Person	Division
John	Production
Bill	Production
John	Research
Mary	Research
Sue	Sales

Tools	
Division	Tool
Production	hammer
Research	pen
Research	computer
Admin.	typewriter

Works \bowtie Tools		
Person	Division	Tool
John	Production	hammer
Bill	Production	hammer
John	Research	pen
John	Research	computer
Mary	Research	pen
Mary	Research	computer

- *join contains no tuple that describes Sue,*
- *join contains no tuple that describes the administration or sales division,*
- *join contains no tuple that shows that there is a typewriter.*

□

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Outer Join

Assume $r \in \text{Rel}(\bar{X})$ and $s \in \text{Rel}(\bar{Y})$.

Result format of $r \bowtie s$: \overline{XY}

The outer join extends the “inner” join with all tuples that have no counterpart in the other relation (filled with null values):

Example 3.12 (Outer Join)

Consider again Example 3.11

Works \bowtie Tools		
Person	Division	Tool
John	Production	hammer
Bill	Production	hammer
John	Research	pen
John	Research	computer
Mary	Research	pen
Mary	Research	computer
Sue	Sales	NULL
NULL	Admin	typewriter

Works \Join Tools	
Person	Division
John	Production
Bill	Production
John	Research
Mary	Research

Works \Join Tools	
Division	Tool
Production	hammer
Research	pen
Research	computer

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Formally, the result relation $r \bowtie s$ is defined as follows:

$J = r \Join s$ — take the (“inner”) join as base

$r_0 = r \setminus \pi[\bar{X}](J) = r \setminus (r \Join s)$ — r -tuples that “are missing”

$s_0 = s \setminus \pi[\bar{Y}](J) = s \setminus (r \Join s)$ — s -tuples that “are missing”

$\bar{Y}_0 = \bar{Y} \setminus \bar{X}$, $\bar{X}_0 = \bar{X} \setminus \bar{Y}$

Let $\mu_s \in \text{Dup}(\bar{Y}_0)$, $\mu_r \in \text{Dup}(\bar{X}_0)$ such that μ_s, μ_r consist only of *null* values

$$r \bowtie s = J \cup (r_0 \times \{\mu_s\}) \cup (s_0 \times \{\mu_r\}) .$$

Example 3.12 (Continued)

For the above example,

$J = \text{Works} \Join \text{Tools}$

$r_0 = [\text{“Sue”, “Sales”}]$, $s_0 = [\text{“Admin”, “Typewriter”}]$

$\bar{Y}_0 = \text{Tool}$, $\bar{X}_0 = \text{Person}$

$$\mu_s = \begin{array}{|c|} \hline \text{Tool} \\ \hline \text{null} \\ \hline \end{array} \quad \mu_r = \begin{array}{|c|} \hline \text{Person} \\ \hline \text{null} \\ \hline \end{array}$$

$$r_0 \times \{\mu_s\} = \begin{array}{|c|c|c|} \hline \text{Person} & \text{Division} & \text{Tool} \\ \hline \text{Sue} & \text{Sales} & \text{null} \\ \hline \end{array} \quad s_0 \times \{\mu_r\} = \begin{array}{|c|c|c|} \hline \text{Person} & \text{Division} & \text{Tool} \\ \hline \text{null} & \text{Admin} & \text{Typewriter} \\ \hline \end{array} \quad \square$$

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Left and Right Outer Join

Analogously to the (full) outer join:

- $r \bowtie_{\text{left}} s = J \cup (r_0 \times \{\mu_s\})$.
- $r \bowtie_{\text{right}} s = J \cup (s_0 \times \{\mu_r\})$.

Generalized Natural Join

Assume $r_i \subseteq \text{ Tup}(\bar{X}_i)$.

Result format: $\cup_{i=1}^n \bar{X}_i$

Result relation: $\bowtie_{i=1}^n r_i = \{\mu \in \text{ Tup}(\cup_{i=1}^n \bar{X}_i) \mid \mu[\bar{X}_i] \in r_i\}$

Exercise 3.1

Prove that the Generalized Natural Join is well-defined, i.e., that the order how to join the r_i does not matter.

Proceed as follows:

- Show that the natural join is commutative,
- Show that the natural join is associative,
- ... then complete the proof.

□

Relational Division

Assume $r \in \text{ Rel}(\bar{X})$ and $s \in \text{ Rel}(\bar{Y})$ such that $\bar{Y} \subsetneq \bar{X}$.

Result format of $r \div s$: $\bar{Z} = \bar{X} \setminus \bar{Y}$.

The result relation $r \div s$ is specified as “all \bar{Z} -values that occur in $\pi[\bar{Z}](r)$, with the additional condition that they occur in r together with each of the \bar{Y} values that occur in s ”.

Formally,

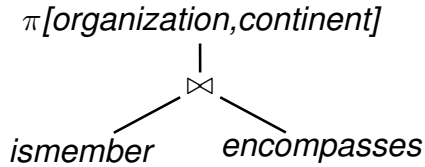
$$r \div s = \{\mu \in \text{ Tup}(\bar{Z}) \mid \mu \in \pi[\bar{Z}](r) \wedge \{\mu\} \times s \subseteq r\} = \pi[\bar{Z}](r) \setminus \pi[\bar{Z}]((\pi[\bar{Z}](r) \times s) \setminus r).$$

- Simple observation: $\pi[\bar{Z}](r) \supseteq r \div s$.
This constrains the set of possible results.
- Often, \bar{Z} and \bar{Y} correspond to the keys of relations that represent the instances of entity types.
- Exercise: the explicit “ $\mu \in \pi[\bar{Z}](r)$ ” in the first characterization looks a bit redundant. Is it? – or why not?

Example 3.13 (Relational Division)

Compute those organizations that have at least one member on each continent:

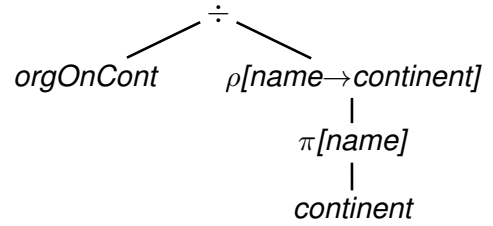
First step: which organizations have (some) member on which continents:



```
SELECT DISTINCT i.organization, e.continent
FROM ismember i, encompasses e
WHERE i.country=e.country
ORDER by 1
```

orgOnCont	
organization	continent
UN	Europe
UN	Asia
UN	N.America
UN	S.America
UN	Africa
UN	Australia
NATO	Europe
NATO	N.America
NATO	Asia
EU	Europe
:	:

Example 3.13 (Cont'd)



$r(\bar{X}), s(\bar{Y}), \bar{Z} := \bar{X} \setminus \bar{Y}$
 $r \div s = \{ \mu \in \text{Typ}(\bar{Z}) \mid \mu \in \pi[\bar{Z}](r) \wedge \{\mu\} \times s \subseteq r \}$
 $\bar{X} = [\text{organization, continent}]$
 $\bar{Y} = [\text{continent}]$
 Thus, $\bar{Z} = [\text{organization}]$.

orgOnCont	
organization	continent
UN	Europe
UN	Asia
UN	N.America
UN	S.America
UN	Africa
UN	Australia
NATO	Europe
NATO	N.America
NATO	Asia
EU	Europe
:	:

$\rho[\text{name} \rightarrow \text{continent}]$ $(\pi[\text{name}](\text{continent}))$
continent
Asia
Europe
Australia
N.America
S.America
Africa

- UN: occurs with each continent in orgOnCont \Rightarrow belongs to the result.
- NATO: does not occur with each continent in orgOnCont \Rightarrow does not belong to the result.
- EU: does not occur with each continent in orgOnCont \Rightarrow does not belong to the result.

Example 3.13 (Cont'd)

Consider again the formal algebraic characterization of the division:

$$r \div s = \{\mu \in \text{Dup}(\bar{Z}) \mid \mu \in \pi[\bar{Z}](r) \wedge \{\mu\} \times s \subseteq r\} = \pi[\bar{Z}](r) \setminus \pi[\bar{Z}]((\pi[\bar{Z}](r) \times s) \setminus r).$$

1. $r = \text{orgOnCont}$, $s = \pi[\text{name}](\text{continent})$, $Z = \text{Country}$.
2. $(\pi[\bar{Z}](r) \times s)$ contains all tuples of organizations with each of the continents, e.g., $(\text{NATO}, \text{Europe})$, $(\text{NATO}, \text{Asia})$, $(\text{NATO}, \text{N.America})$, $(\text{NATO}, \text{S.America})$, $(\text{NATO}, \text{Africa})$, $(\text{NATO}, \text{Australia})$.
3. $((\pi[\bar{Z}](r) \times s) \setminus r)$ contains all such tuples which are not "valid", e.g., $(\text{NATO}, \text{Africa})$.
4. projecting this to the organizations yields all those organizations where a non-valid tuple has been generated in (2), i.e., that have no member on some continent (e.g., NATO).
5. $\pi[\bar{Z}](r)$ is the list of all organizations ...
6. ... subtracting those computed in (4) yields those that have a member on each continent. \square

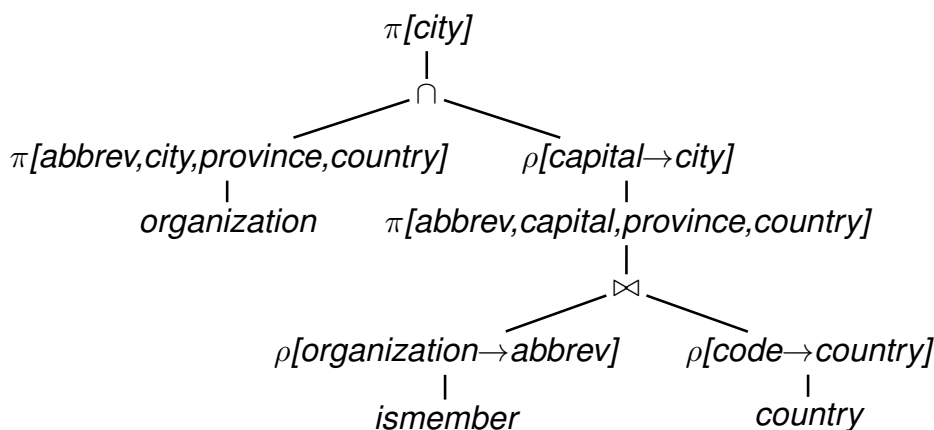
EXPRESSIONS

- inductively defined: combining expressions by operators

Example 3.14

The names of all cities where (i) headquarters of an organization are located, and (ii) that are capitals of a member country of this organization.

As a tree:



Note that there are many equivalent expressions. \square

EXPRESSIONS IN THE RELATIONAL ALGEBRA AS QUERIES

Let $\mathbf{R} = \{R_1, \dots, R_k\}$ a set of relation schemata of the form $R_i(\bar{X}_i)$. As already described, an **database state** to \mathbf{R} is a **structure** \mathcal{S} that maps every relation name R_i in \mathbf{R} to a relation $\mathcal{S}(R_i) \subseteq \text{Tuple}(\bar{X}_i)$

Every algebra expression Q defines a **query** against the state \mathcal{S} of the database:

- For given \mathbf{R} , Q is assigned a **format** Σ_Q (the format of the answer).
- For every database state \mathcal{S} , $\mathcal{S}(Q) \subseteq \text{Tuple}(\Sigma_Q)$ is a relation over Σ_Q , called the **answer set** for Q wrt. \mathcal{S} .
- $\mathcal{S}(Q)$ can be computed according to the inductive definition, starting with the innermost (atomic) subexpressions.
- Thus, the relational algebra has a **functional semantics**.

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SUMMARY: INDUCTIVE DEFINITION OF EXPRESSIONS

Atomic Expressions

- For an arbitrary attribute A and a constant $a \in \text{dom}(A)$, the **constant relation** $A : \{a\}$ is an algebra expression.

$$\Sigma_{A:\{a\}} = [A] \text{ and } \mathcal{S}(A : \{a\}) = A : \{a\}$$

- Every relation name R is an algebra expression.

$$\Sigma_R = \bar{X} \text{ and } \mathcal{S}(R) = \mathcal{S}(R).$$

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SUMMARY (CONT'D)

Compound Expressions

Assume algebra expressions Q_1, Q_2 that define $\Sigma_{Q_1}, \Sigma_{Q_2}, \mathcal{S}(Q_1)$, and $\mathcal{S}(Q_2)$.

Compound algebraic expressions are now formed by the following rules (corresponding to the algebra operators):

Union

If $\Sigma_{Q_1} = \Sigma_{Q_2}$, then $Q = (Q_1 \cup Q_2)$ is the **union** of Q_1 and Q_2 .

$\Sigma_Q = \Sigma_{Q_1}$ and $\mathcal{S}(Q) = \mathcal{S}(Q_1) \cup \mathcal{S}(Q_2)$.

Difference

If $\Sigma_{Q_1} = \Sigma_{Q_2}$, then $Q = (Q_1 \setminus Q_2)$ is the **difference** of Q_1 and Q_2 .

$\Sigma_Q = \Sigma_{Q_1}$ and $\mathcal{S}(Q) = \mathcal{S}(Q_1) \setminus \mathcal{S}(Q_2)$.

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INDUCTIVE DEFINITION OF EXPRESSIONS (CONT'D)

Selection

For a selection condition α over Σ_{Q_1} , $Q = \sigma[\alpha](Q_1)$ is the **selection** from Q_1 wrt. α .

$\Sigma_Q = \Sigma_{Q_1}$ and $\mathcal{S}(Q) = \sigma[\alpha](\mathcal{S}(Q_1))$.

Projection

For $\emptyset \neq \bar{Y} \subseteq \Sigma_{Q_1}$, $Q = \pi[\bar{Y}](Q_1)$ is the **projection** of Q_1 to the attributes in \bar{Y} .

$\Sigma_Q = \bar{Y}$ and $\mathcal{S}(Q) = \pi[\bar{Y}](\mathcal{S}(Q_1))$.

Natural Join

$Q = (Q_1 \bowtie Q_2)$ is the **(natural) join** of Q_1 and Q_2 .

$\Sigma_Q = \Sigma_{Q_1} \cup \Sigma_{Q_2}$ and $\mathcal{S}(Q) = \mathcal{S}(Q_1) \bowtie \mathcal{S}(Q_2)$.

Renaming

For $\Sigma_{Q_1} = \{A_1, \dots, A_k\}$ and $\{B_1, \dots, B_k\}$ a set of attributes,

$Q = \rho[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k](Q_1)$ is the **renaming** of Q_1

$\Sigma_Q = \{B_1, \dots, B_k\}$ and $\mathcal{S}(Q) = \rho[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k](\mathcal{S}(Q_1))$.

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Example

Example 3.15

Professor(PNr, Name, Office), *Course*(CNr, Credits, CName)

teach(PNr, CNr), *examine*(PNr, CNr)

- For each professor (name) determine the courses he gives (CName).

$$\pi [\text{Name}, \text{CName}] ((\text{Professor} \bowtie \text{teach}) \bowtie \text{Course})$$

- For each professor (name) determine the courses (CName) that he teaches, but that he does not examine.

$$\begin{aligned} & \pi [\text{Name}, \text{CName}] ((\\ & \quad (\pi [\text{Name}, \text{CNr}] (\text{Professor} \bowtie \text{teach})) \\ & \quad \setminus \\ & \quad (\pi [\text{Name}, \text{CNr}] (\text{Professor} \bowtie \text{examine})) \\ & \quad) \bowtie \text{Course}) \end{aligned}$$

Simpler expression:

$$\pi [\text{Name}, \text{CName}] ((\text{Professor} \bowtie (\text{teach} \setminus \text{examine})) \bowtie \text{Course}) \quad \square$$

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EQUIVALENCE OF EXPRESSIONS

Algebra expressions Q, Q' are called **equivalent**, $Q \equiv Q'$, if and only if for all structures S , $S(Q) = S(Q')$.

Equivalence of expressions is the basis for **algebraic optimization**.

Let $\text{attr}(\alpha)$ the set of attributes that occur in a selection condition α , and Q, Q_1, Q_2, \dots expressions with formats X, X_1, \dots

Projections

- $\bar{Z}, \bar{Y} \subseteq \bar{X} \Rightarrow \pi[\bar{Z}](\pi[\bar{Y}](Q)) \equiv \pi[\bar{Z} \cap \bar{Y}](Q)$.
- $\bar{Z} \subseteq \bar{Y} \subseteq \bar{X} \Rightarrow \pi[\bar{Z}](\pi[\bar{Y}](Q)) \equiv \pi[\bar{Z}](Q)$.

Selections

- $\sigma[\alpha_1](\sigma[\alpha_2](Q)) \equiv \sigma[\alpha_2](\sigma[\alpha_1](Q)) \equiv \sigma[\alpha_1 \wedge \alpha_2](Q)$.
- $\text{attr}(\alpha) \subseteq \bar{Y} \subseteq \bar{X} \Rightarrow \pi[\bar{Y}](\sigma[\alpha](Q)) \equiv \sigma[\alpha](\pi[\bar{Y}](Q))$.

Joins

- $Q_1 \bowtie Q_2 \equiv Q_2 \bowtie Q_1$.
- $(Q_1 \bowtie Q_2) \bowtie Q_3 \equiv Q_1 \bowtie (Q_2 \bowtie Q_3)$.

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EQUIVALENCE OF EXPRESSIONS (CONT'D)

Joins and other Operations

- $\text{attr}(\alpha) \subseteq \bar{X}_1 \cap \bar{X}_2 \Rightarrow \sigma[\alpha](Q_1 \bowtie Q_2) \equiv \sigma[\alpha](Q_1) \bowtie \sigma[\alpha](Q_2)$.
- $\text{attr}(\alpha) \subseteq \bar{X}_1, \text{attr}(\alpha) \cap \bar{X}_2 = \emptyset \Rightarrow \sigma[\alpha](Q_1 \bowtie Q_2) \equiv (\sigma[\alpha](Q_1)) \bowtie Q_2$.
- Assume $\bar{V} \subseteq \overline{X_1 X_2}$ and let $\bar{W} = \bar{X}_1 \cap \overline{V X_2}$, $\bar{U} = \bar{X}_2 \cap \overline{V X_1}$.
Then, $\pi[\bar{V}](Q_1 \bowtie Q_2) \equiv \pi[\bar{V}](\pi[\bar{W}](Q_1) \bowtie \pi[\bar{U}](Q_2))$;
(Note: unary operations bind stronger than binary operations)
- $\bar{X}_2 = \bar{X}_3 \Rightarrow Q_1 \bowtie (Q_2 \text{ op } Q_3) \equiv (Q_1 \bowtie Q_2) \text{ op } (Q_1 \bowtie Q_3)$ where $\text{op} \in \{\cup, \setminus\}$.
(distributivity of \bowtie wrt. \cup and \setminus)
Note the similarity to the arithmetic term algebra: $n \cdot (a \pm b) = (n \cdot a) \pm (n \cdot b)$

Exercise 3.2

Prove some of the equalities (use the definitions given on the "Base Operators" slide). □

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EXPRESSIVE POWER OF THE ALGEBRA

Transitive Closure

The transitive closure of a binary relation R , denoted by R^* is defined as follows:

$$\begin{aligned}
 R^1 &= R \\
 R^{n+1} &= \{(a, b) \mid \text{there is an } s \text{ s.t. } (a, x) \in R^n \text{ and } (x, b) \in R\} \\
 R^* &= \bigcup_{1.. \infty} R^n
 \end{aligned}$$

Examples:

- $\text{child}(x, y)$: $\text{child}^* = \text{descendant}$
- flight connections
- flows_into of rivers in MONDIAL

Theorem 3.2

There is no expression of the relational algebra that computes the transitive closure of arbitrary binary relations r . □

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EXAMPLES

Time to play. Perhaps postpone examples after comparison with SQL (next subsections)

Aspects

- join as “extending” operation (cartesian product – “all pairs of X and Y such that ...”)
- equijoin as “restricting” operation
- natural join/equijoin in many cases along key/foreign key relationships
- relational division (in case of queries of the style “return all X that are in a given relation with all Y such that ...”)