



Yesterday: "Symbolic Reasoning"

Background: Philosophical logics, mathematical logics, model theory aspects: human reasoning about properties of the logic).

Each logic, and thus also First-Order Logic provides a framework that can be used for symbolic reasoning:

FOL Formulas are strings, FOL reasoning are algorithms that work on their parse trees.

=> symbolic reasoning: all about Syntax, not Semantics

Formulas are evaluated wrt. first-order-logic structures/interpretations

Syntax: the symbols used for writing formulas:

- \* logical symbols:  $\wedge, \exists, \dots$
- \* variables:  $x, y, \dots$
- \* depending on the application: predicate symbols and function symbols, "signature"  $\Sigma$

for mondial:  $\Sigma = \{Country, City, name, hasCapital, \dots\}$

FOL Structure:  $\mathcal{S} = (I, \mathcal{D})$

$\mathcal{D}$  is the domain ... the things in the real world.

$I$  maps the symbols from  $\Sigma$  to the domain ...

Example: our "real-world-application" contains a (green) frog, and strings and numbers:

$\mathcal{D} = \{ \text{🐸} \} \cup \text{Strings} \cup \text{Numbers} \dots$

Signature to talk about the frog and its properties: (1-ary and 2-ary predicates and constant symbols)

$\Sigma = \{ \text{Frog}/1, \text{Green}/1, \text{name}/2, \text{bob}/c0 \}$

Interpret the symbols in OUR structure/model  $\mathcal{S}$  (=current situation):

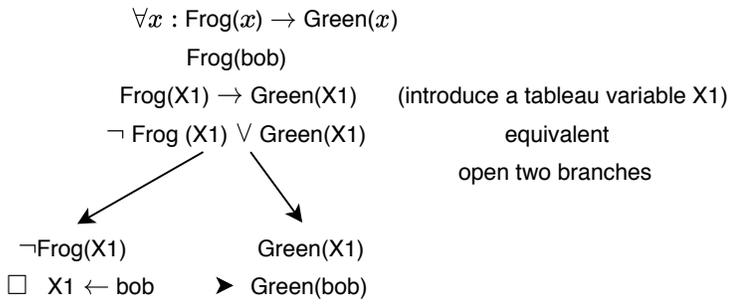
- $I(\text{bob}) = (\text{🐸})$  (an element from  $\mathcal{D}$ )
- $I(\text{name}) = \{ (\text{🐸}, \text{"Bob"}), \dots \}$  (a set of 2-tuples over  $\mathcal{D}$ )
- $I(\text{Frog}) = \{ (\text{🐸}), \dots \}$  (a set of 1-tuples over  $\mathcal{D}$ )

Knowledge base  $\mathcal{K}$ : all frogs are green.

$\forall x : \text{Frog}(x) \rightarrow \text{Green}(x)$

Our  $\mathcal{S}$  must be a model of  $\mathcal{K}$ :

Tableau calculus: what can we derive?



=> conclusion by reasoning: bob must be green in our  $\mathcal{S}$

$\Rightarrow I(\text{Green}) \supseteq I(\text{bob})$

$I(\text{Green}) \supseteq \{ (\text{🐸}) \}$

$I$  practically is a database, containing unary and binary tables:

(note: DB is only on the syntax level, so bob <-> 🐸 )

name	
bob	"Bob"
:	:

Frog
bob

(the constant bob/c0 is like an object identifier)